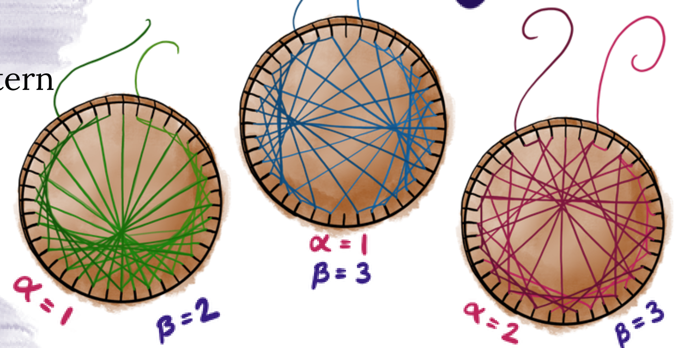
Make Your Own Mathematical String Art

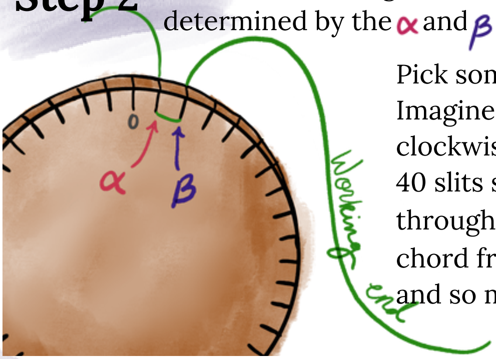
Step 1

Choose a pattern to make.



Step 2

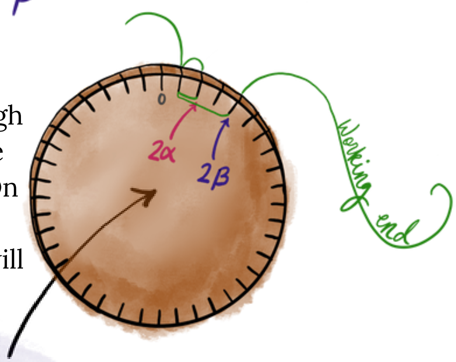
You will be making chords of the circle with endpoints determined by the α and β for your chosen pattern.



Pick some slit on the disk to label 0. Imagine numbering the rest of the slits clockwise around the circle. There are 40 slits so they will have numbers 0 through 39. For each $\boldsymbol{p}$, we will make a chord from $\boldsymbol{p}*\alpha$ to $\boldsymbol{p}*\beta$. Start with $p = 1$ and so make a chord from α to β.

Step 3

Next, weave the working end back through the 2α slit and connect it to the 2β slit.

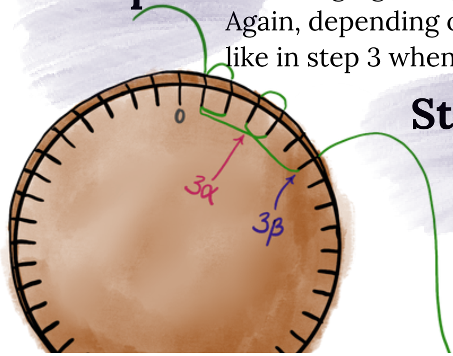


Depending on α and β, you might not be able to physically weave all chords in order because it would require you to pass through the same slit twice. To fix this, either weave the chords out of order or “cheat” a little. On step 3, you can connect $2\alpha - 1$ and 2β and then get back on track after this chord. It will make a minimal difference in the final product.

The second chord starts at $2\alpha-1$ instead of 2α in order to make its creation physically possible.

Step 4

Continue moving the α endpoint a distance of α and the β endpoint a distance of β each time you make a chord. Pull the string tight as you weave to make straight lines. Again, depending on α and β, you might run into an issue like in step 3 when you get to 3α and 3β.



Step 5

When both endpoints return to 0, you are done. Tie off the ends and enjoy your work!.

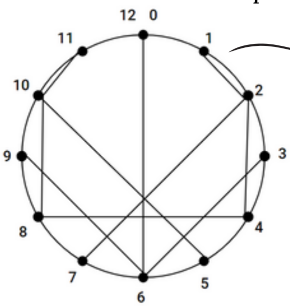
(optional) Step 6

Use a needle to thread a string through the cardboard and turn your creation into an ornament.

Remark

It is very easy to undo and redo the pattern. You can explore other patterns by trying different values for α and β — even negative values work!

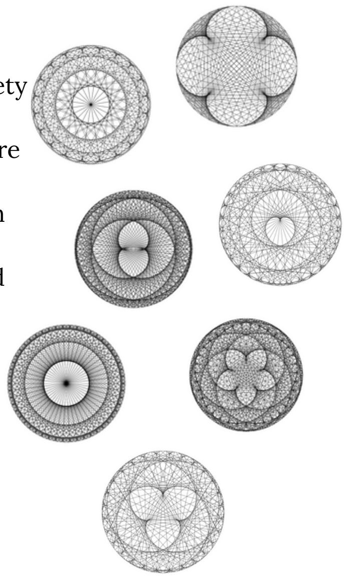
For example, try $\alpha = 1$ and $\beta = 14$. What do you notice?



When $\alpha = 1$, these patterns can be realized as *modular multiplication tables*. Start with $\boldsymbol{m}$ evenly spaced points around a circle, label the points 0 to $\boldsymbol{m-1}$. Then choose a *multiplier* $\boldsymbol{a}$ and connect every point $\boldsymbol{p}$ to $\boldsymbol{a*p \bmod m}$ with a chord of the circle. Such objects are very popular in the recreational math world.

Modular multiplication tables display a large variety of patterns and it is not always clear *why* we get a certain pattern from numbers $\boldsymbol{m}$ and $\boldsymbol{a}$. However, there is a deep connection between these objects and a continuous time dynamical system which can be seen topologically. This change of perspective reveals the mystery. To learn more about this story, you can read my paper on the subject.

To draw more of these patterns for yourself, download my code or use the webapp linked below.



my code **webapp** **paper**

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